

## Solutions

1. Evaluate the function  $f(x) = (\cos x)^{\sin x}$  and its derivative  $f'(x)$  at  $x = 0$ .

(A) 0, 0      (B) 1, 1      (C) 0, 1      (D) 1, 0      [Correct] (E) undef, undef

**Tags: Derivative**

D,

$$\begin{aligned}f &= e^{\sin x \ln \cos x} \\f' &= e^{\sin x \ln \cos x} (\cos x \ln \cos x + \sin x (-\tan x)) \\f'(0) &= 0 \\f(0) &= 1\end{aligned}$$

2. A bee is flying at 0.5 m/s at a distance 2 meters towards a concave thin lens with focal length 0.5 meter along the principal axis of the lens. Find the velocity of the image of the bee.

(A) 0.056 m/s towards the lens.  
(B) 0.056 m/s away from the lens.  
(C) 0.02 m/s towards the lens.  
(D) 0.02 m/s away from the lens. [Correct]  
(E) 0

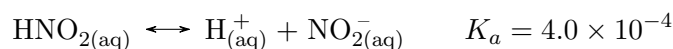
**Tags: Optics**

D,

$$\begin{aligned}\frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\d_i &= \frac{d_o f}{d_o - f} \\d'_i &= f \frac{d_o - f - d_o}{(d_o - f)^2} d'_o \\d'_i &= (-0.5) \frac{-(-0.5)}{(2 - (-0.5))^2} 0.5 = -0.02 \text{ m/s}\end{aligned}$$

negative  $d_i$  means the image is on the same side of the object; negative  $d'_i$  means  $d_i$  decreases which means the image is flying away from the lens. Mistreating the lens as convex will yield 0.056 m/s.

- 3.



Based on the given information, estimate the percent ionization of  $\text{HNO}_2$  in a 1.0 M  $\text{HNO}_{2(\text{aq})}$  solution.

(A) 0.00040%  
(B) 0.020%

- (C) 0.040%
- (D) 0.40%
- (E) 2.0% [Correct]

**Tags: Chemical Equilibrium**

E,

Let the concentration of  $H^+$  be  $x$ , from equilibrium definition we have

$$\begin{aligned}x^2/(1-x) &= K_a \\x^2 &\approx K_a \\x &= \sqrt{K_a} = 0.02\end{aligned}$$

Therefore percent ionization is approximately 2.0 %.

4. The Moon phase on 2/20/2025 is Last Quarter. Estimate the moon phase on 2/27/2025.

- (A) New Moon [Correct]
- (B) First Quarter
- (C) Full Moon
- (D) Last Quarter
- (E) Waning Crescent

**Tags: Moon Phase**

A,

Given Moon's synodic period is 29.5 days (from new moon to new moon). Each quarter takes roughly 7 days. Therefore we estimate the Moon is a New Moon on 2/27th.

5. What's printed when the following program is executed?

```
#include <stdio.h>
int main(void){
    int i = 0;
    int a = i++;
    int b = ++i;
    printf("%d %d", a, b);
    return 0;
}
```

- (A) 0 0
- (B) 1 1
- (C) 1 2
- (D) 0 1
- (E) 0 2 [Correct]

**Tags: Evaluation**

E,

$a=i++$  evaluates assignment before increment;  $b=++i$  evaluates increment before assignment.

6. How many pairs of real solutions for  $(x,y)$  can be found from the two equations

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$
$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

0

**Tags: Algebra, Geometry**

Observe the two conic curves carefully, the first one is a horizontal hyperbola with semi-major axis 3, and the second one is a vertical ellipse with semi-minor axis 2. They won't intersect.