Solutions

1. Evaluate the function $f(x) = (\cos x)^{\sin x}$ and its derivative $f'(x)$ at $x = 0$.

   (A) 0, 0 **(B)** 1, 1 **(C)** 0, 1 **(D)** 1, 0 [Correct] **(E)** undef, undef

   Tags: Derivative

   D,

$$\begin{aligned} f &= e^{\sin x \ln \cos x} \\ f' &= e^{\sin x \ln \cos x}(\cos x \ln \cos x + \sin x(-\tan x)) \\ f'(0) &= 0 \\ f(0) &= 1 \end{aligned}$$

2. A bee is flying at 0.5 m/s at a distance 2 meters towards a concave thin lens with focal length 0.5 meter along the principal axis of the lens. Find the velocity of the image of the bee.

   (A) 0.056 m/s towards the lens.

   (B) 0.056 m/s away from the lens.

   (C) 0.02 m/s towards the lens.

   (D) 0.02 m/s away from the lens. [Correct]

   (E) 0

   Tags: Optics

   D,

$$\begin{aligned} \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\ d_i &= \frac{d_o f}{d_o - f} \\ d_i' &= f\frac{d_o - f - d_o}{(d_o - f)^2} d_o' \\ d_i' &= (-0.5)\frac{-(-0.5)}{(2-(-0.5))^2} 0.5 = -0.02\,\text{m/s} \end{aligned}$$

   negaive d_i means the image is on the same side of the object; negative d_i' means d_i decreases which means the image is flying away from the lens. Mistreating the lens as convex will yield 0.056 m/s.

3. $$\text{HNO}_{2(\text{aq})} \leftrightarrow \text{H}^+_{(\text{aq})} + \text{NO}^-_{2(\text{aq})} \qquad K_a = 4.0 \times 10^{-4}$$

   Based on the given information, estimate the percent ionization of HNO_2 in a 1.0 M $\text{HNO}_{2(\text{aq})}$ solution.

   (A) 0.00040%

   (B) 0.020%

(**C**) 0.040%

(**D**) 0.40%

(**E**) 2.0% [Correct]

Tags: Chemical Equilibrium

E,

Let the concentration of H^+ be x, from equilibrium definition we have

$$\begin{aligned} x^2/(1-x) &= K_a \\ x^2 &\approx K_a \\ x &= \sqrt{K_a} = 0.02 \end{aligned}$$

Therefore percent ionization is approximately 2.0 %.

4. The Moon phase on 2/20/2025 is Last Quarter. Estimate the moon phase on 2/27/2025.

(**A**) New Moon [Correct]

(**B**) First Quarter

(**C**) Full Moon

(**D**) Last Quarter

(**E**) Waning Cresent

Tags: Moon Phase

A,

Given Moon's synodic period is 29.5 days (from new moon to new moon). Each quarter takes roughly 7 days. Therefore we estimate the Moon is a New Moon on 2/27th.

5. What's printed when the following program is executed?

```
#include <stdio.h>
int main(void){
    int i = 0;
    int a = i++;
    int b = ++i;
    printf("%d %d", a, b);
    return 0;
}
```

(**A**) 0 0 (**B**) 1 1 (**C**) 1 2 (**D**) 0 1 (**E**) 0 2 [Correct]

Tags: Evaluation

E,

a=i++ evaluates assignment before increment; b=++i evaluates increment before assignment.

6. How many pairs of real solutions for (x,y) can be found from the two equations

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$
$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

0

Tags: Algebra, Geometry

Observe the two conic curves carefully, the first one is a horizontal hyperbola with semi-major axis 3, and the second one is a vertical ellipse with semi-minor axis 2. They won't intersect.