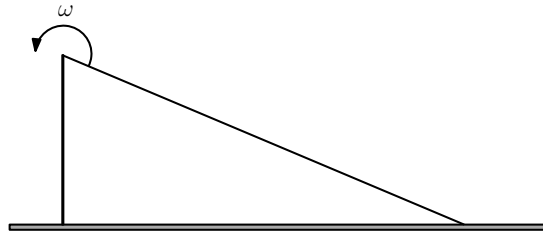


- How many different solutions including both real and complex numbers can be found from $x^{2025} = 1$?
(A) 1 (B) 2 (C) 2024 (D) 2025 (E) 2026
- Find the area of a triangle whose sides are 6, 7, and 9.
(A) 20 (B) 21 (C) 22 (D) $\sqrt{440}$ (E) $\sqrt{420}$
- A math club has 20 members and 3 officers: President, Vice President, and Treasurer. However, one member, Bob, hates another member, Alice. How many ways can we fill the offices if Bob refuses to serve as an officer if Alice is also an officer.
(A) 5931 (B) 6012 (C) 6135 (D) 6611 (E) 6732
- A lamp is mounted on top of a 3-meter-high post. The light from the lamp starts by shining on the ground 10 meters away from the base of the post. The lamp slowly rotates at a speed of $\omega = 0.1$ radians per second. How fast is the spot of light moving along the ground?



- (A) 1.15 m/s (B) 2.34 m/s (C) 3.63 m/s (D) 4.51 m/s (E) 5.92 m/s
- 3 cities A, B, and C are connected by bi-directional roads in between each pair of cities. Suppose the traffic flow follows a probability-based transition such that at each time step, cars follow the proportions given by the matrix P , where:

$$P = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \\ 0.3 & 0.7 & 0 \end{bmatrix}$$

e.g. $P_{1,2} = 0.6$ means a car at from city B has a probability 0.6 to travel to city A, $P_{3,2} = 0.7$ means a car from city B has a 0.7 probability to travel to C.

If the initial distribution of cars at the cities A, B, and C is given by:

$$X_0 = \begin{bmatrix} 300 \\ 400 \\ 200 \end{bmatrix}$$

Find the distribution of cars in cities A, B, and C after one transition step.

- (A) 320, 250, 370
(B) 300, 260, 380
(C) 350, 230, 360
(D) 300, 270, 370
(E) 320, 270, 350

6. Rank the time it takes for the following round objects to roll down a ramp without slipping from the same height. Assume all objects are uniform in density.
- (I) A cylinder made of aluminum.
 - (II) A cylinder made of copper.
 - (III) A hollow ball made of aluminum.
 - (IV) A solid ball made of copper.
- (A) $I = I > III > IV$
 - (B) $I = III > II = IV$
 - (C) $IV > III > II > I$
 - (D) $III > I = II > IV$
 - (E) Cannot be determined from the given information since no radius is given.
7. A refrigerator operates inside a garage during winter. The ambient temperature in the garage is -10 celsius, while the interior of the refrigerator is maintained at -5 celsius. Determine the coefficient of performance (COP) of the refrigerator.
- (A) -2
 - (B) 2
 - (C) -53.6
 - (D) -82.1
 - (E) This setup is impossible.
8. Consider a uniformly charged sphere with total charge Q and radius R . If the electric potential is defined to be 0 at infinity, what is the electric potential at the center of the sphere?
- (A) $\frac{3}{8\pi\epsilon_0} \frac{Q}{R}$
 - (B) $\frac{1}{4\pi\epsilon_0} \frac{Q}{R}$
 - (C) $\frac{3}{4\pi\epsilon_0} \frac{Q}{R}$
 - (D) $\frac{1}{2\pi\epsilon_0} \frac{Q}{R}$
 - (E) $\frac{1}{3\pi\epsilon_0} \frac{Q}{R}$
9. Bob looks into a mirror and notices that the reflection of his nose appears half the actual size of his nose. His eyes are positioned above his nose, confirming that the image is upright. Given that the mirror has a focal length magnitude of 20 cm, determine how far Bob's nose is from the mirror.
- (A) 5 cm (B) 10 cm (C) 15 cm (D) 20 cm (E) 25 cm
10. Twin brothers Adam and Bob are separated when they are 20 years old. Bob hops onto a spacecraft moving at constant velocity $3/5c$ away from Adam. According to Adam, how old is Bob when Adam is 40 years old? c is the speed of light in vacuum. Assume Bob hops onto the spacecraft instantly.
- (A) 36 (B) 38 (C) 40 (D) 42 (E) 44

11. A 2.00 gram sample of iron reacts with oxygen in a combustion process. The resulting products consist of 30.0% iron(II) oxide and 70.0% iron(III) oxide by mass. Determine the volume of oxygen gas consumed under STP conditions.
- (A) 125 mL (B) 536 mL (C) 629 mL (D) 934 mL (E) 1318 mL
12. In a chemical reaction, the concentration of reactant A is measured over time. When the logarithm of the concentration of A is plotted against time t , the graph shows a linear relationship. What is the order of the reaction with respect to A?
- (A) 0th order
(B) 1st order
(C) 2nd order
(D) 3rd order
(E) 4th order
13. A 0.01 M HCl solution is used to titrate an unknown weak base in a 250 mL flask. At the midpoint of the titration, when 30 mL of titrant has been added, the pH of the solution is found to be 10. What is the base dissociation constant of the weak base?
- (A) 10^{-4}
(B) 2.5×10^{-4}
(C) 6.3×10^{-5}
(D) 8.2×10^{-5}
(E) 4.1×10^{-6}
14. In a galvanic cell, two metal electrodes generate a standard reduction potential of 1.2 V. Each half-reaction involves the transfer of 2 electrons. What is the cell potential when the reaction quotient Q is 7.389 at 25 °C?
- (A) 0.83 V (B) 1.14 V (C) 1.2 V (D) 1.36 V (E) 1.41 V
15. What's the molecular geometry of chlorine trifluoride?
- (A) Trigonal bipyramidal
(B) Seasaw
(C) T-Shaped
(D) Pyramidal
(E) Bent
16. Estimate the distance to a star whose parallax angle is 2 milliarcsecond?
- (A) 100 pc (B) 200 pc (C) 300 pc (D) 400 pc (E) 500 pc
17. For a person living at 40 degrees north, what's the maximum altitude of the Sun throughout the year?
- (A) 63.5° (B) 68.5° (C) 73.5° (D) 78.5° (E) 83.5°

18. Which 3 stars constitute the summer triangle. They are very bright stars in the night's sky.
- (A) Procyon, Rigel, and Capella
(B) Deneb, Vega, and Altair
(C) Arcturus, Spica, and Regulus
(D) Capella, Aldebaran, and Betelgeuse
(E) Canopus, Sirius, and Rigel
19. Rigel's apparent magnitude is 1.5. If we move Rigel 10 times further away from the Earth, what's the new apparent magnitude of Rigel?
- (A) -0.1 (B) 0.2 (C) 3.6 (D) 5.8 (E) 6.5
20. A spacecraft orbits around the Earth at an altitude 400 km above the surface, and a full revolution takes 93 minutes. An astronaut on a space walk neglects safety precautions and tosses away a spanner at a speed of 1 m/s directly towards the Earth. You may assume that the Earth is a sphere of uniform density. Estimate the distance between the spanner and the spacecraft after 93 minutes.
- (A) 1 m (B) 10 m (C) 100 m (D) 1000 m (E) 3000 m
21. Which line in the C code will generate a run time error?
- ```
1 #include <stdio.h>
2
3 int main() {
4 int arr[5] = {1, 2, 3, 4, 5};
5 int *ptr = arr;
6
7 printf("%d\n", *ptr);
8
9 ptr[5] = 10;
10
11 return 0;
12 }
```
- (A) 4      (B) 5      (C) 7      (D) 9      (E) 11
22. Find the line with a syntax error in the python code,
- ```
1. def add_numbers(a, b):
2.     result = a + b
3.     return result
4.
5. number1 = 10
6. number2 = 20
7.
8. total = add_numbers(number1, number2)
9. print(f"The sum is: {total}")
10.
11. if total > 10
12.     print("Total is greater than 10")
```
- (A) 1 (B) 3 (C) 8 (D) 9 (E) 11

23. Which statement below is correct?

- (A) Sorting algorithms with the same time complexity perform equally well
- (B) All while loops can be rewritten into for loops in C language
- (C) Floating-point numbers are precise
- (D) More threads always make a program faster
- (E) Hashing makes data completely secure

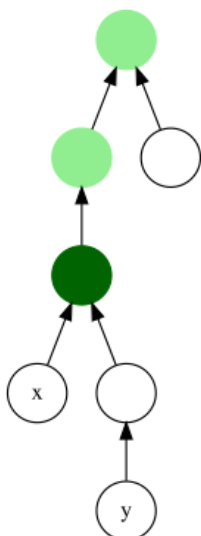
24. You are given an unsorted array of n distinct integers, where each element is in the range $[1, n]$. You are allowed to modify the array in-place with $O(1)$ extra space and $O(n^2)$ time complexity. What is the most efficient way to place each element in increasing order?

- (A) Use Merge Sort, then iterate once to check if all elements are at the correct index.
- (B) Use a HashSet to store seen numbers and rearrange the elements accordingly.
- (C) Use a cycle sort-based approach, swapping elements until each one is in the correct position.
- (D) Apply counting sort, but with an additional array to store counts.
- (E) None of the above

25. You are given a binary tree with n nodes, where each node contains a unique integer value. You need to find the lowest common ancestor (LCA) of two given nodes u and v in the tree. Which of the following algorithms provides the most efficient solution in $O(n)$ worst-case time complexity?

In graph theory and computer science, the lowest common ancestor (also called least common ancestor) of two nodes v and w in a tree or directed acyclic graph (DAG) T is the lowest (i.e. deepest) node that has both v and w as descendants, where we define each node to be a descendant of itself (so if v has a direct connection from w , w is the lowest common ancestor).

In the figure shown, the lowest common ancestor of the nodes x and y is marked in dark green. Other common ancestors are shown in light green. DFS represents depth first search where you visit the nodes as deep as possible before visiting their siblings. BFS represents breadth first search where you visit the sibling nodes of a node first before diving deeper into the subtree of the node. In a binary search tree (BST), each node always has two child nodes with the value in the left node less than or equal to the value in the right node.

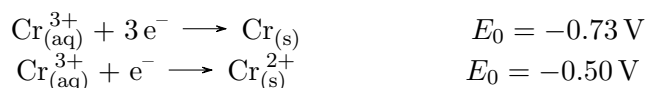


- (A) Perform a DFS traversal for each node separately and store paths, then compare paths to find the LCA.
- (B) Convert the binary tree into a binary search tree (BST) and use the BST property to find LCA efficiently.
- (C) Use a recursive DFS approach, where each node returns the LCA if it finds both target nodes in its subtree.
- (D) Use an adjacency list representation of the tree and perform BFS from both nodes until a common ancestor is found.
- (E) None of the above

26. a, b, c are the roots of the polynomial $f(x) = x^3 - 7x^2 + 14x - 8$. How much is $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$? Write your answer as a decimal number with 5 significant figures.
27. A block of mass 1 kg is placed on a frictionless wedge of mass 3 kg that is free to slide on a horizontal surface. The wedge has an incline angle 30° with respect to the horizontal. The block is released from rest at the top of the incline.

Find the acceleration of the wedge in m/s^2 . Do not specify the unit in your answer, only the numerical part of the answer.

28. What's the standard reduction potential in Volts for $\text{Cr}_{(\text{aq})}^{2+}$ to $\text{Cr}_{(\text{s})}$? The following half reactions are given,



Use the table of constants provided for this problem. Do not specify the unit in your answer, only the numerical part of the answer.

29. White Dwarfs are stars in their last stage of life that are prevented from collapsing only by the electron degeneracy pressure. This pressure is an outward one exerted by the electrons inside the star, which are fermions subject to the Pauli exclusion principle. We can find its value by the following formula, which is derived from the theory of fermion gases:

$$p_{\text{electron}} = \frac{\hbar^2}{5m_e} (3\pi^2)^{2/3} n^{5/3}$$

where n is the number density of electrons in the star. This pressure balances the inward gravitational pressure, which is given by

$$p_{grav} = \frac{GM^2}{5VR}$$

where M is the mass of the star, V is the volume of the star and R is the radius of the star.

If the star contains nuclei with atomic number 6 and mass number 12, what is the density value of the white dwarf in kg/m^3 ?

Use the table of constants provided for this problem. Do not specify the unit in your answer, only the numerical part of the answer.

30. Farmer John's cows have been experimenting with different sorting techniques. They recently came up with a new sorting algorithm (which does not necessarily work correctly), they're unsure how it behaves for different inputs. Your task is to analyze the algorithm and determine the final output of the given program.

```
def mysterious_sort(arr):
    n = len(arr)
    for i in range(n):
        min_idx = i
        for j in range(i+1, n):
            if (arr[j] % 10, arr[j]) < (arr[min_idx] % 10, arr[min_idx]):
                min_idx = j
        arr[i], arr[min_idx] = arr[min_idx], arr[i]
    return arr
```

```
# Read input
N = int(input())
A = list(map(int, input().split()))

# Process the list
sorted_A = mysterious_sort(A)
```

```
# Print output
print(" ".join(map(str, sorted_A)))
```

Input Format The input consists of:

An integer N ($2 \leq N \leq 100$) representing the number of elements in an array.

A list of N distinct integers between -1000 and 1000.

What's the 10th number printed from the program when the following $N = 20$ and list is used with the algorithm? The first number printed is -10.

20

34 -27 89 12 -45 73 56 -91 20 67 41 -38 99 -10 82 -53 31 15 -66 48